FLOW ABOUT LOW-CONDUCTIVITY FRACTURES IN AN UNSATURATED POROUS MEDIUM

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ABSTRACT Many flows observed in subsurface rock formations are unsaturated flows through a medium containing fractures. In cases where the fracture voids are small and nonconnective, one finds that little flow occurs within those voids. Under these conditions, the fractures act as barriers which impede background flow in the underlying matrix. From a computational point of view, discrete modeling of these fractures in flow simulations is impractical not only due to the small size of the fractures, but also because the fractures often represent singularities in the flow field. In general, the background flow through unsaturated porous media is highly nonlinear because of the strong dependence of the hydraulic conductivity on the suction head. For analytical studies, a two-parameter exponential model for the conductivity has been used in conjunction with a Kirchhoff transformation to cast the nonlinear governing equation into a linear form which is more amenable to solution. In this research, simple problems were analyzed using a boundary element numerical implementation of this transformation approach which was applied to characterize the influence of fractures upon the effective hydraulic conductivity of the medium.

KEYWORDS: boundary element, effective conductivity, fracture, porous media, unsaturated flow

INTRODUCTION

The numerical simulation of fluid flows through fractured porous media has many applications in a wide variety of environmental and hazardous waste related issues which include ground water quality, chemical transport, and the design of nuclear waste repositories. In these applications, a knowledge of the flow behavior is required in order to reliably assess both the short-term and long-term impact of contaminant transport.

Geologically-fractured media are composed of intact blocks of matrix material bounded by thin regions referred to as fractures. Intimate contact across the fracture occurs locally through a distribution of asperities. Aside from the volume occupied by asperities, the fractures are essentially void of material. Although the fractures often contain loose material and are coated with mineral precipitates, these effects are ignored in this study.

Unlike a homogeneous porous medium in which steady and transient flow can often be characterized using a single set of material parameters, flow in a fractured medium often takes on more of a dual nature. That is, the flow behavior is partially attributed to the porous matrix and partially to the fractures. While Muskat [1] had mentioned the concept of overlapping media in the context of fractured media, some years passed until the idea was further developed into dual-porosity models of flow in

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fractured porous media. Early interest in these models of fracture flow seem to have arisen from a need to estimate extraction rates of petroleum products from geologic media in which fractures provide primary pathways of fluid flow.

Since all rock formations contain fractures, it is not surprising that the flow in these fractured media have been the subject of many analytical and numerical studies. From a computational point of view, discrete modeling of these fractures in flow simulations is impractical not only due to the small size of the fractures, but also because the fractures might represent singularities in the flow field. Barenblatt, *et al.* [2], noted that despite the irregular structure of fractured rock, it was conceivable that a mean behavior of the medium could be obtained by averaging the local behavior on a scale larger than several matrix blocks. In this study they presented the notion of different flow velocities and pressures in the fractures and the matrix. Barenblatt, *et al.*, also suggested a possible means of coupling between the matrix and fracture flow as well as simplifications which would permit analytical solutions.

Warren and Root [3] also concluded that in some cases there would be an equalization of pressure in the matrix and in the fractures as a result of flow between the two regions termed pseudo-steady interporosity flow. They provided additional information about parameters used to distinguish flow in a fractured medium from that in a homogeneous medium. The formulation of Warren and Root was then applied to transient radial flow in a well.

Later wellbore studies [4-6] provide a more theoretical basis to interporosity flow by treating the case of transient matrix block to fracture flow. From Moench's [6] work, it can be concluded that the assumption of pseudo-steady interporosity flow is valid when precipitates line the fracture.

The previously cited work applies strictly to saturated flow in fractured porous media, but many of the flows observed in rock formations occur under unsaturated flow conditions. In the case of saturated flow, a distinction was made between pressure in the fractures and in the matrix. Because timescales and flow rates in unsaturated flow are much slower than in saturated flow, it may not be necessary to make such a distinction in unsaturated flow. Owing to pressure differences, the overall flow distribution between the fractures and the matrix in a saturated medium depends upon the interaction of flow in the two regions. However in an unsaturated porous medium, one often finds that the flow characteristics are determined by the effective saturation of the matrix in addition to gravity and capillary forces.

In accordance with the Young-Laplace relation, the fracture voids may be viewed as large pores so that capillary moisture is preferentially retained in the pores of the matrix material. Provided that vapor transport is negligible then in cases where the fracture voids are small and non-connective, one finds that flow might not even occur within the fracture voids themselves, but instead through the matrix blocks and across asperities in the fractures. Thus under these conditions, the fracture voids do not participate in the background fluid flow, but instead act as barriers which impede flow in the underlying matrix.

Because the fractures are ubiquitous at all scales in geologic materials, successful modeling of fluid flows in porous media relies heavily on the ability to include the effects of the fractures. Rather than try to model the individual fractures, a continuum approach is often used to account for the net flow behavior on a larger scale through the use of effective properties for the medium. This approach would be useful for problems in which the length scale of the fluid flow being studied is larger than the scale of the fractures. In this work, it is assumed that the net effects of the fractures are periodically distributed within the medium and that these effects can be captured by lumping the void regions of the fracture.

The effective property approach has been used extensively in the analysis of porous medium saturated flow problems, e.g., in layered shales, Begg and King [7], and in a fractured medium, Rasmussen, *et al.* [8]. Using a boundary element method (BEM) approach, Rasmussen, *et al.*, predicted effective vertical conductivities for both a two- and three-dimensional fractured medium by considering fractures of finite extent. They also found that for highly impermeable fractures, the predicted effective conductivity for the three-dimensional medium was smaller than that for the two-dimensional case although flow area blockage is less than in three-dimensions. Noting that in unsaturated flow the hydraulic conductivity varied with the matrix potential Martinez, *et al.* [9], used a finite-difference approach to study the effective conductivity in a two-dimensional periodically-fractured unsaturated porous medium and obtained results consistent with those of Rasmussen, *et al.* at the impermeable fracture limit.

A two-dimensional boundary element method has been previously developed for the analysis of steady quasilinear flow in an unsaturated porous medium composed of a single material in arbitrary geometries [10]. The two-dimensional boundary element formulation was extended to allow the analysis of multi-zoned regions of dissimilar materials [11]. In this paper, the BEM methodology is applied in studies of effective conductivity similar to Martinez, *et al.* [9]. The current work serves as a preliminary study for an analysis corresponding to the three-dimensional saturated flow problem addressed by Rasmussen but for unsaturated flow conditions.

BOUNDARY ELEMENT FORMULATION

Because the hydraulic conductivity in unsaturated porous media depends upon the moisture content, the governing equation for flow, Richard's equation, is nonlinear. The moisture content can be related to the capillary pressure head, so that the hydraulic conductivity can alternatively be expressed in terms of the capillary pressure head ψ. In this research, we consider a two-parameter exponential model for the hydraulic conductivity, K, given by

$$K(\psi) = K_s \exp(\alpha\psi), \quad \psi \le 0, \tag{1}$$

where K_s is the saturated conductivity and α is a material constant. While this model does not represent certain materials well over the entire range of pressures encountered in unsaturated flows, the model has nevertheless been successfully applied to a large class of problems [12-14]. Using a Kirchhoff transformation in conjunction with the exponential model of hydraulic conductivity from a reference state, ψ_o yields the definition of the Kirchhoff potential,

$$\Phi = \int_{\psi_o}^{\psi} K(\eta)\,d\eta. \tag{2}$$

Thus, the variation of conductivity over the applicable pressure range is represented by a single variable. Furthermore, this transformation facilitates a quasilinearization of the nonlinear governing equation. Differentiation of Equation 2 and substitution into the steady state Richard's equation [1], yields a linear equation in terms of the Kirchhoff potential Φ,

$$\nabla^2\Phi - \alpha\frac{\partial\Phi}{\partial z} = 0. \qquad (3)$$

As a result of the transformation, for exp $\alpha\psi_o$ << 1 pressure is transformed as

$$\Phi = K_s \exp(\alpha\psi)/\alpha, \qquad (4)$$

and that the Darcy flux transforms as

$$q = \nabla\Phi - \alpha\Phi e_z. \qquad (5)$$

It follows from Equations 1 and 4 that the transformation could also be written in terms of a relative permeability, $\phi = \exp(\alpha\psi) = \Phi\alpha/K_s$, which provides a measure of the medium conductivity relative to that of the saturated state.

Using Green's second identity, the boundary value problem for Φ can be reformulated as a boundary integral equation given by

$$\frac{1}{2}\Phi(x) + \int_\Gamma \frac{\partial G(x,y)}{\partial n}\Phi(y)d\Gamma(y) = \int_\Gamma G(x,y)(-q_n(y))d\Gamma, \qquad (6)$$

where Γ is the boundary of the domain, $q_n = -\partial\Phi/\partial n + \alpha\Phi n_z$ is the flux normal to the boundary Γ, $n_z = n \bullet e_z$, and $G(x, y)$ is the free-space Green's function given by

$$G(x,y) = \frac{1}{2\pi}\exp\left(\frac{\alpha z}{2}\right)K_o\left(\frac{\alpha r}{2}\right), \qquad (7)$$

where $r = |x - y|$, the distance between the field point $x(x_1, x_2)$ and the source point $y(y_1, y_2)$, and $z = x_2 - y_2$, the spatial distance between the two points in the direction of gravity.

Numerical solutions of the boundary integral equation are obtained by discretizing the domain boundary Γ into elements and approximating the boundary values of Φ(y) and q_n(y) within each element using polynomial nodal basis functions. By collocating Equation 6 at selected points, one obtains a system of linear algebraic equations which can formally be written as

$$[A_{ij}]\{\Phi_j\} = [B_{ij}]\{q_j\}, \qquad (8)$$

where Φ_j and q_j represent the values of Φ and q_n, respectively, at the global collocation nodes. Since either Φ_j, q_j, or a linear combination of the two is specified at each node, Equation 8 can be rearranged and solved to determine the remaining unknowns. Once the boundary unknowns have been determined, the net inflow and outflow boundary flux can be obtained by numerical integration. The integrated boundary flux must later be used in computing an effective conductivity for a heterogeneous medium.

The previous formulation is applicable for analysis of flow in a single material, but general applications usually deal with several materials. This capability is necessary for studying flows in layered porous media as well as flows with permeable inclusions. In a numerical model, a discontinuous representation of a discrete fracture can be obtained by viewing the fracture as two overlapping no-flow boundaries. Unfortunately, within the boundary element methodology, this type of representation renders the system of equations, Equation 8, singular. This difficulty can be overcome using a technique originally used to model sheet piles in dams [15] by which the medium is treated as two different zones, both consisting of the same material but each containing only one face of the fracture.

The single zone formulation applies directly to individual homogeneous zones in a multi-zoned region, but requires the use of interface conditions to provide problem closure. In the fracture problem, we impose these interface conditions across areas in physical contact. For zoned homogeneous regions, a boundary integral equation is written for each region in terms of the Kirchhoff potential Φ_j for that region. From physical considerations, there is an equality of both the normal flux and pressure at the interface between zoned regions. In the quasilinear formulation, the flux condition remains unchanged, but the pressure condition now corresponds to a discontinuity in the Kirchhoff potential at interfaces between dissimilar materials. Formally stated, the pressure condition between two homogeneous regions, Ω_i and Ω_j, is

$$\left(\frac{\alpha_i \Phi_i}{K_{si}}\right)^{1/\alpha_i} = \left(\frac{\alpha_j \Phi_j}{K_{sj}}\right)^{1/\alpha_j}. \tag{9}$$

As in the single material case, the discretized boundary integral equations for the kth subregion can again be written as

$$\left[A_{ij}^k\right]\left\{\Phi_j^k\right\} = \left[B_{ij}^k\right]\left\{q_j^k\right\}, \tag{10}$$

but due to the interfacial pressure boundary condition, Equation 9, the assembled system of equations is now nonlinear. The solution strategy adopted here is to employ an iterative method. Values of the interfacial potentials $\Phi_i^{k,\text{int}}$ are assumed known in a single subregion, from which Equation 9 then implies knowledge of $\Phi_j^{k,\text{int}}$ in the adjoining region. Using these values of the interface potential, each subregion k can then be analyzed independently to yield estimates of the unknown normal flux q_{n^k} at the interface. Application of the interface flux boundary condition between regions Ω_i and Ω_j produces a nonlinear residual equation at each interface node,

$$R(\Phi) = q_n\left(\Phi_i\right)_i + q_n\left(\Phi_j\right)_j. \tag{11}$$

To provide a meaningful solution, this set of nonlinear equations must be solved for the unknown interface potentials which minimize the residual equations

$$\|R(\Phi)\| \le \varepsilon \tag{12}$$

to some desired tolerance, ε. In this work, the nonlinear residual equations are solved using an approach previously employed by Martinez [11], a modification of the Powell hybrid method. The previously described formulation was implemented as a FORTRAN program and then used to solve several relevant problems.

VERIFICATION

Because we will later model the fractured medium as two material regions, it is appropriate to verify the ability of the code to model two coupled material zones. Verification of numerical solutions to problems in unsaturated flow is sometimes difficult because analytic solutions of nonlinear problems are somewhat limited. This is particularly true in the case of multi-dimensional and multi-zoned problems. The numerical implementation of the previous quasilinear formulation for layered media was verified by first analyzing a one-dimensional problem in an homogeneous medium using two overlying layers having identical material properties for various flux and pressure boundary conditions. For the problem analyzed, excellent agreement was obtained between the numerical solutions

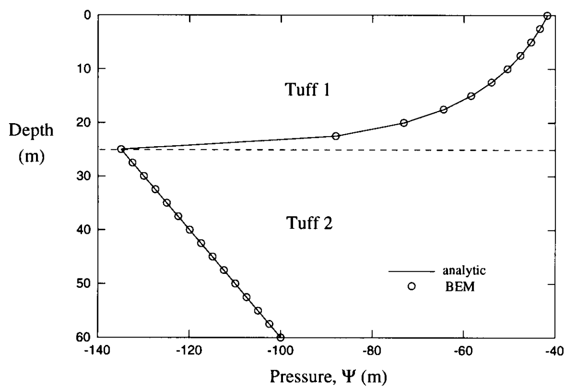

FIGURE 1. PRESSURE DISTRIBUTION IN THE LAYERED MEDIA VERSUS DEPTH.

and analytic solutions discussed in Martinez and McTigue [16].

Further verification of the formulation for multi-zoned regions was obtained by considering the problem of steady, downward infiltration into a two-layered medium. Although this problem is strictly one-dimensional, it does serve to demonstrate the ability of the code to handle several materials. The problem studied considers a downward unsaturated flow of magnitude 1.0 mm/yr to a constant pressure of -100 m through two rock layers. Model parameters for each layer, K_s and α, were selected as representative values for two different tuffs and are shown in Table 1.

The two-layer problem was analyzed using the boundary element method, and the numerical results were compared to an analytic solution. Martinez and McTigue [16] have given an analytic solution for the layered media problem in terms of the Kirchhoff potential. However, because the Φ solution range varies over several orders of magnitude it is more useful to compare the pressure solutions. The analytical and numerically-computed pressure distribution for the two-layer problem is shown in Figure 1 and demonstrates good quantitative agreement between the two results. In the figure, we also see that, as in many unsaturated flow problems, there can be abrupt changes of pressure over relatively short distances. A comparison of the two solutions indicates that the relative error in the BEM pressure solution is about 1%. Based upon the favorable results obtained in the verification problem, we expect that the numerical procedure is appropriate for the fracture problem.

EFFECTIVE CONDUCTIVITY OF A PERIODICALLY-FRACTURED MEDIUM

We now examine the problem of determining the effective conductivity of unsaturated fractured porous media. Given specific boundary conditions, we define the effective conductivity of a fractured medium as the ratio of net flux in the fractured medium to the net flux in an unfractured medium at the same boundary.

It is assumed that the net effects of the fractures are periodically distributed within the medium and that the overall effects on the flow can be accounted for by lumping the void portions of the fracture into discrete

TABLE 1. MATERIAL PARAMETERS FOR TWO-LAYER INFILTRATION PROBLEM.

Unit	Thickness (m)	K_s (m/s)	α (1/m)
Tuff 1	25.0	9.7×10^{-12}	3.82×10^{-2}
Tuff 2	35.0	3.9×10^{-7}	1.71×10^{-2}

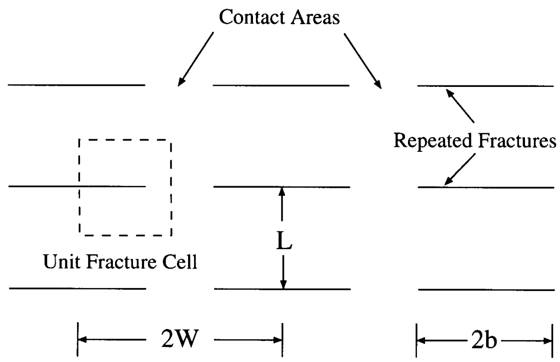

FIGURE 2. CONCEPTUAL MODEL OF PERIODIC FRACTURE FIELD.

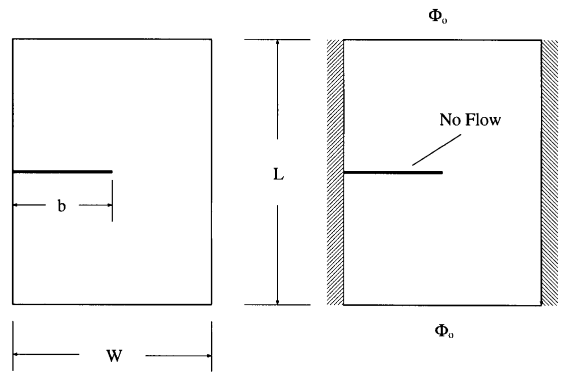

FIGURE 3. UNIT FRACTURE CELL MODEL.

and impenetrable boundaries embedded in the matrix. The idealized fractured medium, shown schematically in Figure 2, is characterized in terms of the vertical fracture spacing, L, the horizontal fracture spacing, $2W$, and the fracture void width, $2b$. It is assumed that the entire fractured medium is subject to a unit gradient condition in which the downward flux is constant. Under this flow condition, the behavior of a single unit fracture cell reflects the flow characteristics of the entire fractured medium. It is this unit cell which will be analyzed to determine the effective conductivity of the medium.

Fractures in this medium are horizontally oriented and since horizontally adjacent cells are identical, there can be no horizontal flow between neighboring cells. We further note that under the unit gradient condition, changes in matrix or pressure potential are offset by corresponding changes in gravity potential so that matrix potential and the Kirchhoff potential are periodic in the vertical direction. The unit fracture cell along with the boundary conditions for the fracture problem are shown in Figure 3.

The previous description of the fractured medium applies equally well to both a saturated and unsaturated medium. Likewise we expect that asymptotic limit of the unsaturated flow problem for low α corresponds to the saturated flow problem, which is governed by the Laplace equation in the piezometric head. The corresponding saturated flow problem in a periodically-fractured medium has previously been investigated by Martinez, *et al.* [9]. Using a conformal mapping technique they were able to show that the effective conductivity of the medium was

$$K_{eff} = \frac{K(\Psi)}{K(\Psi_o)} = \left[1 - \frac{4}{\pi}\frac{W}{L}\ln\left[\cos\left(\frac{\pi}{2}\frac{b}{W}\right)\right]\right]^{-1} \tag{13}$$

where Ψ_o is a reference pressure. While this analytic solution cannot be computed directly by our boundary element code, it does represent the limit of effective conductivity for low α.

From dimensional analysis, we find that the appropriate dimensionless parameters for this problem are the ratios of horizontal fracture width to horizontal fracture spacing (b/W), vertical to horizontal fracture spacing (L/W), and the sorptive number, αL. Based upon these dimensionless parameters, we then fix b/W and analyze unit fracture cells of three different L/W for a range of αL using the boundary element code. This permits us

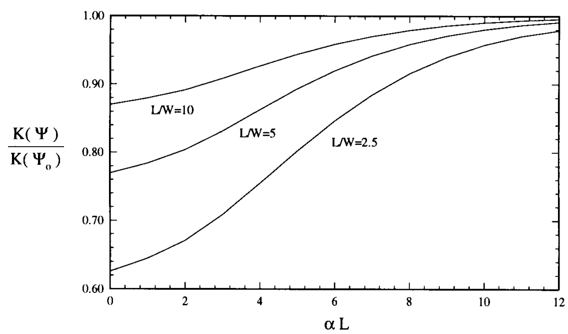

FIGURE 4. EFFECTIVE HYDRAULIC CONDUCTIVITY VERSUS αL FOR VARIOUS L/W RATIOS (b/W = 0.8).

to examine general trends in the effective conductivity.

The effective conductivity for b/W = 0.8 and three different L/W ratios are given in Figure 4 for a range of αL. In this figure, the lower limit values of effective conductivity (αL = 0) correspond to the analytical results of Equation 13 whereas the remaining results are values computed from the BEM results. The computed values of conductivity for each L/W ratio are consistent with the earlier results of Martinez, *et al.* [9] in that they transition smoothly toward the $\alpha L = 0$ analytical result. Overall, these results also indicate that a wide variation of effective conductivity is possible for any given L/W ratio.

DISCUSSION

A two-dimensional boundary element formulation for determining the effective conductivity of fractured porous media has been developed. Using the BEM methodology a simple study has been carried out to determine the effective hydraulic conductivity of a periodically-fractured porous medium. In this study, computational cells representing different vertical fracture density and varying percentages of asperities are used to determine the effective conductivity as a function of conductivity ratio of an otherwise homogeneous medium. For the cases examined, the effective conductivity displays upper limits corresponding gravity-dominated flow and lower limits where the effects of capillarity are important. Results presented herein serve primarily as a demonstration that the previous results of Martinez, *et al.* [9], can be obtained by another numerical method, a method in which more general geometries are more easily addressed. Further work should exploit the advantage of the alternative BEM method to extend the present work to three dimensions and to evaluate the effective conductivity for cases in which the flow is not normal to the fractures.

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