MODELING OF HEAVY METAL TRANSPORT AND SOIL EROSION IN SURFACE RUNOFF

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ABSTRACT A physically-based model was developed for rainfall, runoff, erosion, and solute transport processes on land surfaces under time-varying rainfall events. Kinematic wave approximation was used to describe the overland flow dynamics. Erosion equation was represented as a first-order reaction, with the reaction rate being represented by soil erodibility. The solute transport equations were based on a convective-dispersive model that incorporates rate-limited mass transfer through a laminar boundary layer at the soil surface/runoff water interface. The model has been validated using data available in literature. Results for the erosion and solute transport processes were validated separately. Experiments are planned for complete model validation.

KEYWORDS: modeling, heavy metal transport, soil erosion, overland flow

INTRODUCTION

Contamination of surface water by heavy metals poses a serious environmental and health risk in regions where coal and heavy metals were mined previously and subsequently abandoned. Drainage from such abandoned coal and heavy metal mine lands can result in contamination of surface water with heavy metals such as cadmium, lead, zinc, chromium, and copper. The release and migration of nutrients, pesticides, and other chemicals from agricultural lands is also a threat to the quality of surface waters. Heavy metals at or near the soil surface can be transformed to overland flow in solution form by the mixing of rainwater with soil solution, dissolution of the heavy metal partly present in solid form, desorption of an adsorbed or absorbed chemical from the soil and residues in place, and desorption of the chemical from eroded sediment. The amount of heavy metal desorbed from eroded sediment is generally much less than the amount desorbed from the soil in place. Furthermore, the heavy metal present on the sediment is likely to have been desorbed partially before the sediment was dislodged. Therefore, for simplicity, the desorption from sediment may be considered as part of the desorption from soil. The mixing and desorption are accelerated by the kinetic energy of rainfall and shear stress produced by overland flow. The infiltration rate of the soil controls the rates of runoff, soil erosion, and chemical transfer.

There have been several modeling efforts in the past to describe the soil erosion process and chemical transport over land surfaces. Some of these are Foster and Meyer [1], Bennett [2], Li [3], and Rose [4]. Studies on solute transport include Ahuja [5], Wallach, *et al.* [6], Wallach and Van Genuchten [7], and Rivlin and Wallach [8]. But there is no model available for simulating both soil erosion and solute transport processes in conjunction with rainfall-runoff. This study presents a model combining these two processes. The erosion component is verified with the results presented by Govindaraju

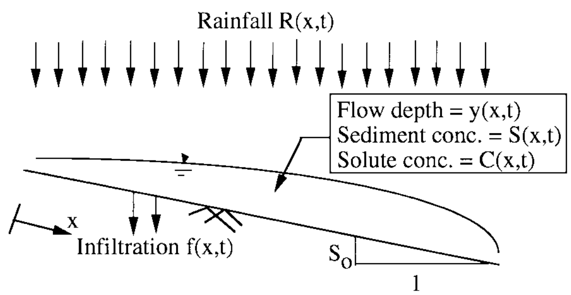

FIGURE 1. SCHEMATIC SKETCH OF OVERLAND FLOW OVER A HILLSLOPE.

and Kavvas [9], and solute transport model performance is compared with results from Rivlin and Wallach [8]. The combined model will be validated with field experimental results that will be conducted later.

GOVERNING DIFFERENTIAL EQUATIONS

Overland flow equations

Figure 1 shows a schematic sketch of overland flow over a hillslope. Govindaraju, *et al*. [10, 11], presented the diffusion wave approximation to the full Saint-Venant equations for overland flow as

$$\frac{\partial y}{\partial t}+\frac{\partial q}{\partial t}=R(x,t)-f(x,t)=i(x,t) \qquad 0 \preceq x \preceq L, \quad t \succ 0\,, \tag{1}$$

$$S_f = S_o - \frac{\partial y}{\partial x}. \tag{2}$$

Here y(x,t) is overland flow depth, q(x,t) is the flow discharge per unit width, L is the length of the hill slope whose slope is S_o, i(x,t) is the net lateral inflow into the overland flow section (and equals to the rainfall R(x,t) minus the infiltration f(x,t)), and S_f is the friction slope. Overland flow can be laminar or turbulent. If velocities and depths of flow are relatively small, the viscosity of the fluid becomes a controlling factor, and the flow is viscous or laminar. Flow is apparently always laminar close to the divide, but the portion of area covered by turbulent flow increases downslope toward the channel because of increasing depth and velocity. Under the kinematic assumption, flow discharge is a function of overland flow depth [12]:

$$q = \alpha \; y^{m+1}. \tag{3}$$

Here α and m are coefficients which depend on whether the flow regime is laminar or turbulent. For laminar flow, m = 2 and $\alpha = 8gs/K_r\upsilon$ where s is the uniform soil surface slope, g is the acceleration due to gravity, υ is the kinematic viscosity of water, and K_r is a parameter related to the soil surface roughness. For turbulent flow (Re > 500), m = 0.5 and $\alpha = C_z s^{1/2}$ where C_z is a Chezy's coefficient or m = 2/3 and $\alpha = s^{1/2}/n$ where n is Manning's coefficient.

Substitution of Equation 3 into Equation 1 gives the kinematic wave equation with one dependent variable:

$$\frac{\partial y}{\partial t}+\alpha\,(m+1)\;y^{m}\;\frac{\partial y}{\partial x}=i(x,t)\,. \tag{4}$$

Sediment transport equations

The soil erosion equations are given by Govindaraju and Kavvas [9] and may be expressed as

$$\frac{\partial (Sy)}{\partial t}+\frac{\partial (Sq)}{\partial x}=\sigma\left[\frac{C_t}{\rho_s}\,(\gamma\, yS_o-\tau_{cr})^{p}\right]+c_1\,\frac{R^{c_2}}{\rho_s}. \tag{5}$$

In Equation 5, S(x,t) is the concentration of the sediment by volume, ρ_s is the mass density of the particles, R is the rainfall rate, C_t and σ are coefficients for determining the erodibility of the soil as a result of sheet erosion, γ is the density of water, τ_{cr} is the critical shear stress, and c_1 and c_2 are constants. The term $\gamma y S_o$ is a measure of the tractive force exerted by the surface flow on the soil particles on the bed. The exponent p varies between 1.0 and 2.5. Foster, *et al.* [13], suggested a value of 1.0 for c_2. The value of c_1 was assumed as 0.85.

Solute transport equations

Chemical transport by overland flow can be described by the following mass conservation equation:

$$\frac{\partial (Cy)}{\partial t} + \frac{\partial (Cq)}{\partial x} = \frac{\partial}{\partial x}\left[D_x \frac{\partial}{\partial x}(Cy)\right] + k(C_s - C) + PC_{sa}\,. \tag{6}$$

Here C(x,t) is the chemical concentration in runoff, and k is the convective mass transfer coefficient that relates solute flux across the soil surface interface to the difference in concentration between the soil solution (C_s) and the runoff water (C). Diffusion coefficient (D_x) and concentration of the chemical adsorbed on to the soil (C_{sa}) are assumed to be zero in this study. The amount of sediment present in the runoff is given by the right hand side of the Equation 5, which is denoted by P in the Equation 6. It is assumed that the soil chemicals are uniformly distributed along the slope and that chemical flux from the soil surface to the overland flow is uniform.

Initial and boundary conditions

The initial and boundary conditions which are applicable to overland flow over steep slopes were discussed by Govindaraju, *et al.* [10], and are used in this study. For a plane of length L, the end conditions are

$$y(x,0) = 0 \qquad 0 \preceq x \preceq L\,, \tag{7a}$$

$$y(0,t) = 0\,, \tag{7b}$$

$$\frac{\partial y}{\partial x}(L,t) = 0\,. \tag{7c}$$

The sediment transport equation requires two boundary conditions and one initial condition for the problem to be well posed. The initial condition for the sediment transport must be one of zero concentration, dictated by the dry initial condition in Equation 7a. Similarly, a zero concentration value at the upstream end of the flow domain is required to coincide with the zero depth condition of the Equation 7b. The downstream boundary condition for the concentration is taken to be similar as the flow condition at that end. Thus we have the following end conditions for the sediment concentration:

$$S(x,0) = 0\,, \tag{8a}$$

$$S(0,t) = 0\,, \tag{8b}$$

$$\frac{\partial S}{\partial x}(L,t) = 0\,. \tag{8c}$$

The initial and boundary conditions for solute transport equation are very similar to sediment transport equations. These are

$$C(x,0) = 0\,, \tag{9a}$$

$$C(0,t) = 0\,, \tag{9b}$$

$$\frac{\partial C}{\partial x}(L,t) = 0\,. \tag{9c}$$

MATHEMATICAL FORMULATION

The finite difference method was adopted to solve the governing differential equations. A fixed grid, implicit central difference scheme with a weighting factor for temporal variation was used. Since overland flow equations are non-linear, Newton’s method was applied at each time step to obtain the flow depths. All three governing differential equations were solved independently, assuming that sediment and chemical concentrations do not influence the flow regime. First, the overland flow equation

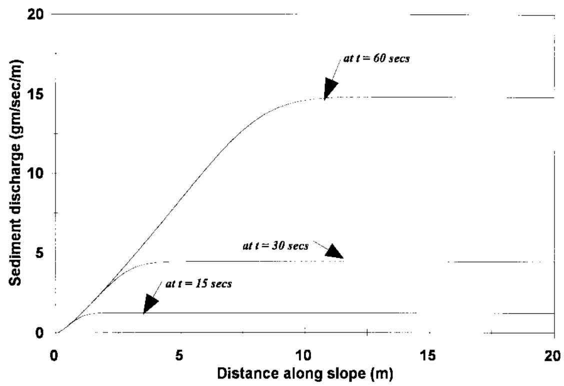

FIGURE 2. SEDIMENT DISCHARGE PROFILES FOR THE RISING PHASE.

was solved to compute flow depth at each spatial grid. Using the flow depth and velocity, sediment transport and solute transport equations are solved.

Writing the finite difference equations at all nodes (N) will result in N non-linear equations for the overland flow process, which can be represented in the matrix form as shown below:

$$[A]\,[y] = [B]\,, \qquad (10)$$

TABLE 1. INPUT PARAMETERS FOR SEDIMENT TRANSPORT MODEL.

Description of input parameter	Value
Length of the hillslope, L (m)	20.0
Slope, S_o	10%
Rainfall intensity, i (mm/hr)	50.0
Rainfall duration, T (min)	10.0
Surface roughness constant, C ($m^{1/2}$/s)	15.0
Soil critical shear stress, (kg/m^2)	0.0015
Soil erodibility constants, s (/m)	10.0
C_t	0.60
Bulk density of solids, r_s (kg/m^3)	1,200.0
Density of water, g (kg/m^3)	1,000.0
Exponent, p	1.50
Constants: c_2	1.00
c_1 (assumed)	0.85

where A is a tridiagonal matrix of size N x N, y is flow depth vector of size N, and B is right hand-side of the difference equation. After solving for y and q, sediment transport and chemical concentration subroutines were called. Both the sediment and chemical transport equations lead to linear tridiagonal matrices. A computer program was written in FORTRAN, and the IMSL subroutine DLSLTR was utilized to solve the tridiagonal system of equations.

DISCUSSION OF RESULTS

Sediment transport model

The input data used for sediment transport model validation is presented in Table 1. A small value of mesh width (100 cm) and time step (5 secs) was selected for this problem. Analytical and numerical solutions for this case were discussed by Govindaraju and Kavvas [9]. Figure 2 presents sediment discharge profiles over the flow plane at different times during the rising phase.

Figure 3 shows the rising and receding limbs of the flow and sediment discharges at the outflow section. Both rising and falling stages showed fast response to rainfall. Steady-state water discharge predicted by the model matched well (~27 cc/sec/m/10.0) with the analytical solution, but steady state sediment discharge obtained was high (~40 gm/sec/m against ~35 gm/sec/m). This may be due to the different assumed parameters for solving sediment transport equations.

Solute transport model

The input parameters for solute transport model are given in Table 2. The concentration of the chemical in solution at the soil surface (C_S) is assumed as 1.0. The water and chemical hydrographs are presented in Figures 4 and 5 for two different values of k. The boundary

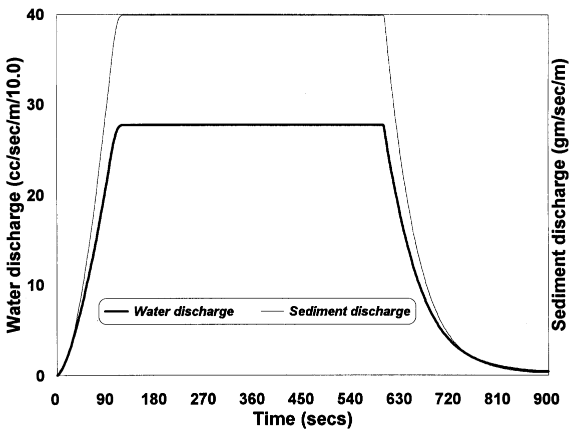

FIGURE 3. RISING AND RECEDING LIMBS OF THE FLOW AND SEDIMENT DISCHARGES AT THE OUTFLOW SECTION.

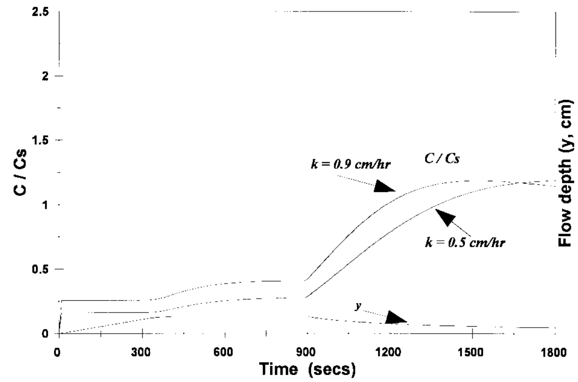

FIGURE 4. OVERLAND FLOW DEPTH AND CHEMICAL HYDROGRAPHS AT THE OUTLET.

conditions used by Rivlin and Wallach [8] are different when compared with this model. They assumed chemical concentration at x = 0 and t = 0 to be C_s to obtain an analytical solution. Because of this, the chemical hydrograph obtained is of different shape (Figure 4 in Rivlin and Wallach [8]). The flow depth hydrograph predicted by the model is similar to their result. Concentration hydrographs increase gradually with flow depth during rising stage. The concentration hydrographs increase gradually with a faster rate during the falling stage of overland flow. Since the soil surface concentration C_s is assumed constant throughout the storm, the variation in overland flow concentration is due to the transfer of dissolved soil chemical to overland flow and its lateral transport toward the slope outlet.

Rainfall termination at t = 900 secs decreases the water and chemical fluxes at the slope outlet, thereby increasing runoff concentration. Since the chemical flux to the overland flow is proportional to the difference between the soil surface concentration, C_s, and the runoff concentration, C, an increase in runoff concentration diminishes this flux. During the falling stage, the soil chemical flux becomes zero and even negative when C becomes larger than C_s. This negative flux follows mathematically from a runoff

TABLE 2. INPUT PARAMETERS FOR SOLUTE TRANSPORT MODEL.

Description of input parameter	Value
Length of the hillslope, L (cm)	10,000.0
Constant, m	3
constant, $a = C\,S_o^j$ ($cm^{1/2}$/hr)	$5.8 * 10^6$
Infiltration rate, f (cm/hr)	0.30
Rainfall intensity, R (cm/hr)	1.60
Rainfall duration, T (min)	15.0
Convective mass transfer coefficient, k (cm/hr)	0.5 & 0.9
Concentration of chemical at soil surface, C_s	1.00

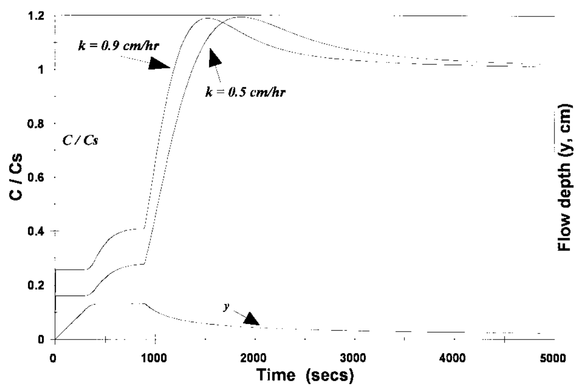

FIGURE 5. OVERLAND FLOW AND CHEMICAL HYDROGRAPHS AT THE OUTLET, RESULTS UP TO t = 5,000 SECS.

concentration that asymptotically approaches $kC_S/(k - i)$ when rainfall is zero.

Overland flow and chemical hydrographs for no infiltration (after t = 900 secs) are presented in Figure 5. The concentration value shoots up immediately after x = 0. This is due to the boundary condition imposed in the model (at x = 0, C = 0 for t > 0). Flow depth profile exhibits the usual shape appearing in many papers and textbooks. It is clear from Figure 5 that the C/C_S curve is asymptotically approaching a value of 1.0 (= C_S) for both the values of k. Since there is no infiltration after rainfall, the rainfall hydrograph has a long receding limb.

CONCLUSIONS

A model was developed for rainfall-runoff-solute transport processes on steep hillslopes subjected to time-varying rainfall events. The model performed well when compared to some existing results in the literature. The performance of the model will be further evaluated after comparing with some more examples in the literature and with other experimental results.

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