CONDUCTIVITY OF SOILS WITH PREFERENTIAL FLOW PATHS

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ABSTRACT Laboratory soil column experiments were conducted to study the distribution of preferential flow paths resulting from removal of fine-size clay particles. These experiments specifically studied the influence of clay (kaolinite) percentage in sand-clay mixtures and the effect of hydraulic gradients on pore evolution. Analysis of the effluent during the experiments indicated that clay particles were removed from the soil column, accompanied by an increase in porosity and hydraulic conductivity. Dye experiments were conducted on the same columns to stain the pathways where clay particle removal occurred. It was observed that pore formation was fairly uniform in some cases, while other cases showed distinct preferential flow path formation. A physically-based model was used to identify a dimensionless parameter, G, which expresses the ratio of detachment and deposition forces at any space-time location. A model, based on equivalent media theory, is proposed to describe the hydraulic conductivity of soils with preferential flow paths. Future work will test the theoretical expressions for conductivity with experimental results, and investigate the relationship between G and the equivalent conductivity for such soils.

KEYWORDS: conductivity, soil, modeling

INTRODUCTION

Detachment of clay particles in either natural or compacted soils is important in predicting the permeabilities of such media and the associated contaminant transport. It is well established in literature that clay particle detachment is possible both by physical and chemical effects induced through pore water flow. While the physical effects are primarily due to creation of a hydrodynamic force (governed by flow velocity) in the porous medium, chemical effects are due to altering the pore fluid characteristics such as electrolyte content and/or pH to dissolve the cementing agents and disperse the particles. The colloidal clay particles provide an additional mobile solid phase for movement of adsorbed contaminants. This study has useful applications in contaminant migration in subsurface and soil remediation issues. To date, many laboratory and field studies [1, 2] have demonstrated the influence of colloidal mobilization on the transport of associated contaminants.

There is hardly any research dealing with determining pore geometry and identification of preferential flow paths in conjunction with fine particle removal. As the fines are dislodged from the soil matrix (and perhaps entrapped later in pores), there is a continuous change in the pore structure, which in turn influences the hydraulic conductivity. This paper has the following objectives: (i) to briefly review the mathematical theory leading to the dimensionless parameter ,G; (ii) to present a theoretical framework from equivalent media theory [3] which evaluates the macroscopic conductivity of soils with preferential flow paths; and (iii) to lay the outline for relating the statistical properties of the random network of flow paths to physical properties like G. It is assumed that the preferential flow paths are the major conduits for water flow and that the rest of the soil matrix does not contribute significantly to movement of the water.

EXPERIMENTAL PROGRAM

The experimental program [4] was designed to etch out dye patterns along preferential flow paths that resulted from application of hydraulic pressure gradients across the soil samples. The program was developed with different clay percentages and different pressure application procedures to study the dye patterns under different conditions of soil type and hydraulic gradients.

The soil samples consisted of compacted sand-clay mixtures. Three different mixtures, differing only in their clay content (5%, 10%, and 15% by weight), were investigated for pore evolution when subjected to different pressure applications. Commercial fine sand (FJ-55, supplied by Mori Screenings), with particle sizes ranging from 0.1 mm to 0.9 mm, was used in all the soil mixtures. The clay used in the mixtures was kaolinite (supplied by Cary Co.) which had a mean particle diameter of 0.77 μm. Appropriate proportions of sand and clay were mixed at a molding water content of 16% and compacted in compaction permeameters. All samples were prepared to a height of 5 cm under the same Proctor compactive effort. The soil samples were then allowed to saturate under a standing head of water until the effluent rate out of the soil column reached a constant value. The standing head of water was very small, and no colloidal removal was observed at this stage in the effluent. After saturation, any excess water standing on the soil sample was removed with a pipette.

A standard dye solution was prepared by mixing 2.5 grams of Evans blue dye powder in 150 ml of water. This dye was added on to the soil sample, and the top cover of the permeameter was closed for controlled pressure application by means of hand pumps (range 0-15 psi). A predetermined pressure was applied to pass the dye solution through the soil sample. As the dye solution passed through the soil, it stained the flow paths. The time taken to pass 150 ml of dye and the pressure application history were recorded.

The dye-stained soil sample was then carefully extruded out of the permeameter by gently tapping on the permeameter walls and was air dried for one hour. As the dye solution rests on the soil sample for a little while before the application of pressure, the top layer of the soil sample (2 to 3 mm) was sectioned off with a sharp knife. The pattern of the dye as it passed through the cross-section was visible and was photographed with a caption identifying the experiment. Three sections, each about 5 mm thick, were sectioned for each soil sample, and their cross-sections were photographed. Similarly, vertical sections were cut and the dye patterns photographed. For more details, refer to Bhargava [5].

STATISTICAL ANALYSIS OF PREFERENTIAL FLOW PATHS

Twenty-four photographs/samples of the vertical soil sections were analyzed. Each photograph contains one network averaging about 100 preferential flow paths each. The photographs were scanned into a computer, and SigmaScan (a PC program) was used to measure the lengths, diameters, and orientation of the preferential flow paths.

The statistical distributions of the lengths, diameters, and orientations were then investigated. Typical frequency distributions are shown in Figures 1 to 3. The lengths and diameters of the preferential flow paths displayed a log normal distribution.

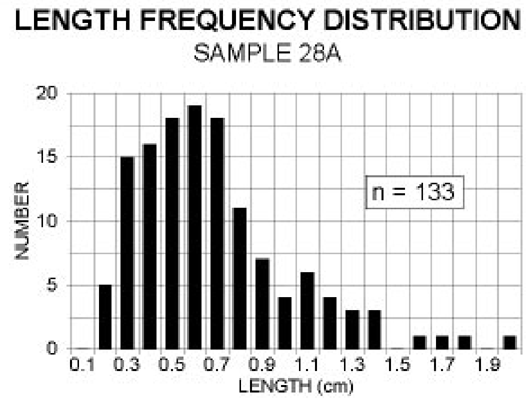

FIGURE 1. TYPICAL LENGTH DISTRIBUTION.

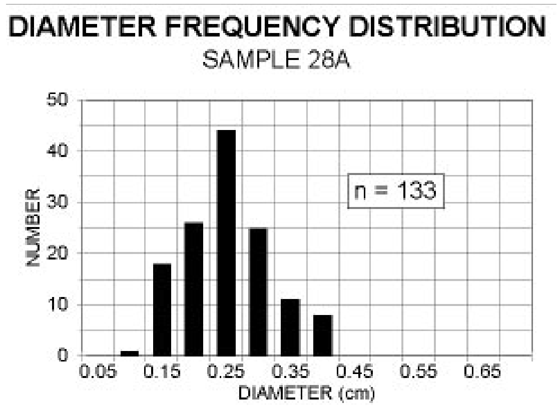

FIGURE 2. TYPICAL DIAMETER DISTRIBUTION.

To test the lengths and diameters for log normality, Kolmogorov and Cramer von Mises statistical tests were conducted [6]. The results are shown below in Tables 1 and 2. For a level of significance taken as $\alpha = 0.15$, the critical values for the Kolmogorov and Cramer von Mises tests are 0.775 and 0.091, respectively. If the value in the table is less than the critical value, the test passes and the hypothesis can be made. For the lengths, 18 of 24 samples passed the Kolmogorov test, and 19 samples passed the Cramer von Mises test. For the diameters, 14 of 24 samples passed the Kolmogorov test, and 18 passed the Cramer von Mises test. Both tests displayed a relatively high power (β), and log normality was concluded.

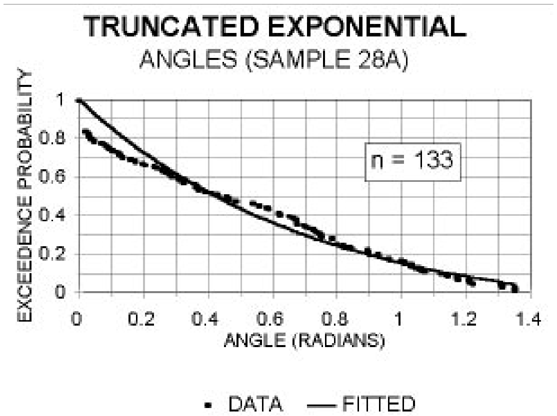

FIGURE 3. TYPICAL ANGLE DISTRIBUTION.

The orientations of the flow paths displayed a truncated exponential distribution. Nonlinear regressions were performed on the orientation data to fit an exponential equation. The truncated exponential distribution is given by

$$g(\Theta) = \left\{ \begin{array}{ll} \dfrac{\alpha e^{-\alpha\Theta}}{1 - e^{\alpha\pi/2}} & , \text{ for } 0 \le \Theta \le \dfrac{\pi}{2} \\ 0 & , \text{ otherwise} \end{array} \right\}. \qquad (1)$$

Chi square tests were then done to test the goodness of fit. The chi square values generally fell within the 85% confidence interval; therefore, a truncated exponential distribution is used to describe the orientations.

Later in the description of the random network model, a hypothesis is made concerning the centers of the preferential flow paths. The hypothesis is the centers follow a Poisson distribution with intensity λ. In this case λ is defined as the number of occurrences of path centers per unit area. From the Poisson distribution, it can be shown that

$$P[R \geq r] = 1 - e^{-\lambda \pi r^2}, \qquad (2)$$

where P is probability of an occurrence within an area, $A = \pi r^2$, and R is a random variable denoting the distance between two closest preferential flow path centers.

The nearest distance to the closest center was then measured for each flow path center. Non-linear regression was used to fit the above equation to the measured data and obtain values of λ. A typical Poisson distribution is shown in Figure 4. Chi square tests are being conducted to test the goodness of fit to support the Poisson hypothesis.

MATHEMATICAL FORMULATION

A physically-based theory that incorporates the mechanisms of particle detachment from the soil matrix, subsequent transportation of the detached colloidal particles in the pore fluid, and possible deposition (or entrapment) [7] was developed. The conservation of fine particles in a porous medium in one-dimension was expressed as

$$\frac{\partial(\eta c)}{\partial t} + \frac{\partial(cq)}{\partial x} - \frac{\partial}{\partial x}\left[hD\frac{\partial c}{\partial x}\right] = R(c)/\rho_s, \qquad (3)$$

TABLE 1. LOG NORMALITY RESULTS FOR LENGTHS.

Sample	Kolmogorov Statistic (D_m)	Cramer Statistic (W^2_m)
22a	0.862	0.118
22b	0.545	0.035
22c	1.147	0.316
22d	0.728	0.072
23a	0.510	0.049
23b	0.369	0.016
23c	0.683	0.071
24a	0.745	0.078
24b	0.687	0.079
24c	0.567	0.038
25a	0.595	0.057
25b	0.559	0.051
25c	0.587	0.050
26b	0.739	0.073
26c	0.722	0.128
26-5a	0.938	0.128
26-5b	0.651	0.071
26-5c	0.908	0.082
27-5a	0.388	0.027
27-5b	1.026	0.252
28a	0.486	0.049
28b	0.813	0.076
29a	0.734	0.071
29b	0.514	0.043

TABLE 2. LOG NORMALITY RESULTS FOR DIAMETERS.

Sample	Kolmogorov Statistic (D_m)	Cramer Statistic (W^2_m)
22a	0.875	0.093
22b	1.960	1.099
22c	0.713	0.075
22d	0.500	0.020
23a	0.795	0.066
23b	0.776	0.059
23c	0.707	0.058
24a	0.504	0.032
24b	0.710	0.084
24c	0.807	0.061
25a	0.564	0.068
25b	0.789	0.059
25c	0.846	0.120
26b	1.226	0.299
26c	0.855	0.073
26-5a	0.595	0.060
26-5b	0.644	0.042
26-5c	0.762	0.070
27-5a	0.557	0.049
27-5b	0.855	0.118
28a	0.759	0.075
28b	0.567	0.041
29a	0.551	0.027
29b	0.689	0.098

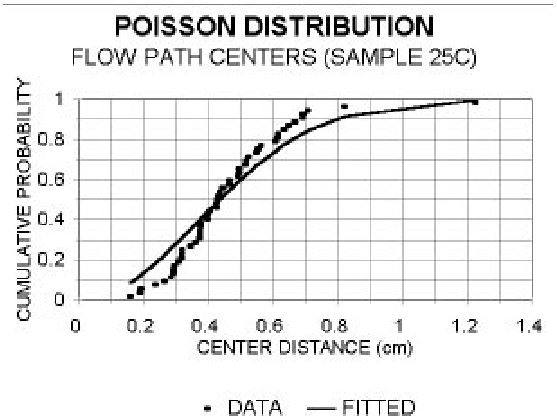

FIGURE 4. TYPICAL POISSON DISTRIBUTION OF FLOW PATH CENTERS.

where $\eta(x,t)$ is the soil porosity, $c(x,t)$ is the volume concentration of clay particles, ρ_s is the mass density of particles, $R(c)$ is the concentration-dependent detachment rate of particles, q is Darcian flux, and D is the Darcian scale dispersion. The dispersion of fine particles was neglected in the present analysis because of the small size of the sample. Experiments were conducted under externally-applied hydraulic gradients, which makes convection the dominant transport mechanism. The particle removal was expressed as

$$R = \rho_s \sigma (T_c/\rho_s - cq), \tag{4}$$

where T_c is the flow transport capacity. This equation suggests that the rate of particle removal is proportional to the difference between the transport capacity of the flow and the actual suspended load in the flow. The constant of proportionality, σ, is particle detachability and is a measure of the intersurface forces binding the particles to the soil. If the carrying capacity of the seepage stream were to be reduced below its existing load in suspension (because of change in pore geometry or lowering of local pore flow velocity), then this excess would be deposited. Therefore the above formulation inherently allows for particle entrapment and clogging.

The transporting capacity of the flow is proportional to the effective shear stress that is applied on the pore walls (applied shear stress minus the critical shear stress of the soil matrix),

$$T_c = \alpha(\tau_w - \tau_{cr}), \tag{5}$$

where α is a proportionality constant determining the erosion rate as a function of the effective shear stress, τ_w is the total applied shear stress, and τ_{cr} is the critical shear stress. A part of the applied stress is due to the shear stress exerted by the seepage water as it moves through the pores and is proportional to the flow velocity. Another component of applied force is due to the time rate of change of pressure gradient across the soil sample. The pressure increase across the soil sample is not instantaneous. The time rate of pressure application induces accelerations in the pore fluid which, in turn, cause additional shearing forces to be exerted on the pore walls. The applied shear stress may be expressed as

$$\tau_w = aq + b\,\frac{dp}{dt}, \tag{6}$$

where a and b are proportionality constants and p is the pressure across the soil sample. The change of porosity was related to change of permeability through the Kozeny-Carman equation. Equations 3 to 6 describe the physical process of transport of inorganic colloidal clay particles. The clay concentration, flow discharge, porosity, and permeability are internally coupled. These equations were solved numerically under appropriate boundary and initial conditions for c, η, and q. The key findings from the modeling study were: (i) accelerations during the initial time period, that are induced by the transient pressure gradients, are responsible for significant detachment of fine particles, and (ii) cumulative removal of particles is dependent on both the magnitude of pore fluid accelerations and the duration of the transient pressure changes across the soil sample.

Based on the above model conceptualization and neglecting dispersion, it is possible to define a dimensionless parameter, G, that represents the ratio of eroding forces to deposition forces [4]. It is mathematically expressed as

$$G = \frac{\alpha}{\rho_s}\left[\frac{a}{c} + \frac{b}{cq}\frac{\partial p}{\partial t} - \frac{\tau_{cr}}{cq}\right]. \tag{7}$$

Equation 7 implies that when G > 1, there is a net removal of particles at that space-time location; for G < 1, there is a net deposition of particles; and G = 1 represents an equilibrium situation with the net removal being zero. This equilibrium is attained when the transporting capacity of the pore fluid is equal to the suspended load. The parameter G incorporates the influence of soil properties, external pressure gradients, and the flow discharge on particle removal and deposition.

Before the above model can be used as a predictive tool, the conductivity of the soil sample needs to be determined to compute the flow rates that are required for computing q in Equations 3 and 6. For this purpose, a random network model is proposed. The theory of random networks has been reviewed in Hestir and Long [3]. The first step is to visualize the network of preferential flow paths in a fashion that will yield the maximum permeability. Then correction factors and modifications can be introduced to make the random model more representative of the actual network of preferential flow paths. The random network of preferential flow paths has the highest vertical conductivity when all the flow paths are aligned vertically and are infinitely long. Assuming laminar conditions, the flow discharge through a preferential flow path of diameter d can be expressed through the Hagen-Poiseuille's equation as

$$q = \frac{\rho g \pi d^4}{128\mu} I_h, \tag{8}$$

where ρ is the density of water, g is gravitational acceleration, μ is the dynamic viscosity, and I_h is the hydraulic gradient. Assuming a two-dimensional random network, where the centers of the preferential flow path segments are distributed in a Poisson fashion with intensity λ, and the diameters of the flow paths follow a random distribution, it can be shown that the maximum intrinsic permeability, k_{max}, can be expressed as

$$k_{max} = \frac{\eta \langle d^4 \rangle}{32 \langle d^2 \rangle} = \frac{\pi}{128} \lambda \langle d^4 \rangle, \tag{9}$$

where η is the porosity of the medium, and <.> denotes the expectation operation. It is assumed that the distribution of the diameters of flow paths is independent of the occurrence process of the flow paths. The maximum conductivity of the medium is expressed as $K_{max} = k_{max}\rho g/\mu$. It can be seen that the expressions for maximum intrinsic permeability only need the second and fourth moments of the distribution of the diameters of the flow paths.

The maximum conductivity for infinitely long flow paths after correction for orientation can be written as

$$K_{max} = \frac{\rho g}{\mu} \cdot \frac{\eta}{32} \cdot \frac{\langle \cos\Theta \rangle^2 \cdot \langle d^4 \rangle}{\langle d^2 \rangle}. \tag{10}$$

In Equation 10, the orientation of flow paths (indicated by Θ which is the angle with respect to vertical) is assumed to be independent of the lengths and diameters of the flow paths.

Since the diameters follow a log normal distribution and the orientations follow a truncated exponential distribution, the expectations of the distributions can be substituted into the equation above and written as

$$K_{\max} = \frac{\rho g}{\mu} \cdot \frac{\eta}{32} \cdot \left\{ \frac{\alpha (e^{-\alpha\pi/2} + \alpha)}{(1+\alpha^2)(1-e^{-\alpha\pi/2})} \right\}^2 \cdot \frac{\exp[4\mu_y + 8\sigma_y^2]}{\exp[2\mu_y + 2\sigma_y^2]}, \qquad (11)$$

where α = parameter of the truncated exponential distribution, μ_y is the mean of the log of the diameters, and σ_y is standard deviation of the log of the diameters.

Equation 11 gives an expression for maximum conductivity for infinitely long preferential flow paths. The similarities between this model and a real soil medium are (i) the porosity is the same, and (ii) the preferential flow path diameters are represented through their distributions.

Since preferential flow paths are not infinitely long, equivalent media theory is used to provide modifications and corrections. To make the random network model more representative of the network of preferential flow paths, we follow the approach of Hestir and Long [3] in defining properties of the random network and finding their analogous representation from equivalent media theory for regular lattices. In regular lattices (also in percolation theory literature), p denotes that probability of a lattice site being conductive and is analogous to the probability of finding a preferential flow path at a particular location. Another property of a regular lattice is the coordination number representing number of connections present at a site. At this time it is convenient to define ζ, the connectivity of the network [3, 8]. It represents the average number of intersections that a preferential flow path has with other paths. Assuming that the preferential flow paths have random lengths, L, whose distribution is independent of occurrences and diameters of the flow path, it can be shown that

$$\zeta = \lambda \langle L \rangle^2 H(\theta), \qquad (12)$$

where $H(\theta)$ represents the orientation correction for the flow paths. Here, θ is the angle which a flow path makes with a reference line (vertical in this instance), and

$$H(\theta) = \int_0^\pi \int_0^\pi \sin|\theta_o - \theta| \, g(\theta) g(\theta_o) d\theta d\theta_o , \qquad (13)$$

where $g(\theta)$ is the distribution of the orientation angles (in radians). For $g(\theta)$ being uniform on $(0,\pi)$, $H(\theta) = 2/\pi$. Equating expressions for average preferential flow path length for random networks with an average length in a corresponding regular lattice [3], results in the following expressions

$$p(\zeta) = \frac{\zeta}{\zeta+2} , \qquad (14)$$

$$z(\zeta) = 4(1- \frac{1}{\zeta}) . \qquad (15)$$

Now using the expression of Kirkpatrick [9] that relates p and z to the ratio of the conductivity of a network to K_{max}, the following expression can be obtained

$$\frac{K}{K_{max}} = \frac{\zeta(\zeta-4)}{\zeta^2-4} . \qquad (16)$$

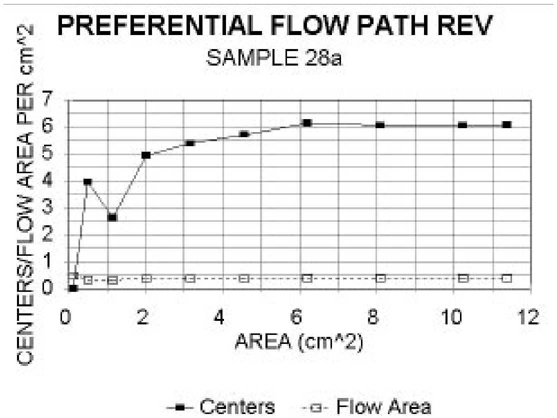

FIGURE 5. TYPICAL SIZE OF REV.

This relates the conductivity of a random network to the maximum conductivity of the network as given by Equation 11. Hestir and Long [3] argue that these expressions need further correction to account for the 'dead-end' flow paths and also those flow paths that do not connect to a conducting portion of the network. Their procedure depends on factoring-off all preferential flow paths less than a threshold cutoff length, to account for the immobile water trapped in these flow paths. A further correction factor may need to be applied to account for dimensionality. K_{max} was derived using three dimensional definitions, but it is only possible to investigate the preferential flow paths in two dimensions.

REPRESENTATIVE ELEMENTARY VOLUME (REV)

An initial investigation of the size of REV was conducted. The centers of the flow paths were counted, and the flow area was calculated for a small unit area on the photograph. The unit area was then increased and the same properties measured. Figure 5 shows the results of this process. The point where the two curves start to level off is considered the REV. In this sample, the REV is around 6 cm^2.

CONCLUSION

This paper presents the concepts and relevant theory that is necessary for finding expressions of conductivity of soils exhibiting preferential flow paths. Parameters of the random network were obtained from the analysis of the dye-photographs. The intensity of occurrence of flow path centers, λ, was obtained through non-linear regression of the Poisson equation. The lengths and diameters of preferential flow paths were found to be log normally distributed, while the orientations of the flow paths followed a truncated exponential distribution.

FUTURE WORK

The theoretical conductivities will be verified with experimentally-obtained conductivities at the end of the experiment. Further validation of the theory will be obtained from Monte-Carlo simulations, wherein networks of preferential flow paths will be generated and the flow across them calculated through a pipe-network model under an imposed gradient of unity. Another use of the Monte-Carlo simulations will be to further investigate of the size of the representative elementary volume (REV) for soils with preferential flow paths.

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